8. Integrimi i funksioneve transcendente

Të njehsohen integralet:

1. $\int xe^x \sin x dx.$ **2.** $\int \frac{e^{3x}dx}{1+2e^x}.$ **3.** $\int \frac{dx}{1+e^{\frac{x}{2}}+e^{\frac{x}{5}}+e^{\frac{x}{10}}}.$

4. $\int \frac{dx}{\sqrt{1+e^x}+\sqrt{1-e^x}}.$ **5.** $\int \ln(\sqrt{1-x}+\sqrt{1+x})dx$

Zgjidhja.

1. Zbatojmë faktin që $\sin x = \frac{e^{ix}-e^{-ix}}{2i}$. Merret

$$\int xe^x \sin x dx = \int xe^x \left(\frac{e^{ix}-e^{-ix}}{2i}\right)dx = \frac{1}{2i}\int x(e^{x(1+i)}-e^{x(1-x)})dx$$

$$= \frac{1}{2i}\left(e^{x(1+i)}\left(\frac{x}{1+i}-\frac{1}{(1+i)^2}\right)-e^{x(1-i)}\left(\frac{x}{1-i}-\frac{1}{(1-i)^3}\right)\right)+C$$

$$= \frac{e^x}{2}(x(\sin x-\cos x)+\cos x)+C.$$

2. Zëvendësojmë: $e^x = u;\ e^{3x}=u^3;\ e^x dx = du.$

$$\int \frac{e^{3x}dx}{1+2e^x} = \int \frac{e^{3x}e^x dx}{(e^x 1+2e^x)} = \int \frac{u^3 du}{u(1+2u)} = \int \frac{u^2}{2u+1}du$$

$$= \int\left(\frac{1}{2}u-\frac{1}{4}+\frac{1}{4}\frac{1}{2u+1}\right)du = \frac{u^2}{4}-\frac{1}{4}u+\frac{1}{4}\ln|2u+1|+C.$$

3. Zëvendësojmë: $e^{\frac{x}{10}} = u \Rightarrow \frac{1}{10}e^{\frac{x}{10}}dx = du;\ e^{\frac{x}{10}}dx = 10du.$ Merret:

$$\int \frac{dx}{e^{\frac{x}{10}}\left(1+e^{\frac{x}{2}}+e^{\frac{x}{5}}+e^{\frac{x}{10}}\right)}=10\int \frac{du}{u\left(1+u^5+u^2+u^1\right)}$$

$$=10\int \frac{du}{u[\left((u+1)+u^2(u^3+1\right)}=10\int \frac{du}{u(u+1)(1+u^2(u^2-u+1))}$$

4. $$I=\int \frac{e^x dx}{e^x(\sqrt{1+e^x}+\sqrt{1-e^x})}=\begin{vmatrix} e^x=u \\ e^x dx=du \end{vmatrix}=\int \frac{du}{u(\sqrt{1+u}+\sqrt{1-u})}$$

$$=\int \frac{\sqrt{1+u}-\sqrt{1-u}}{u(1+u-1+u)}du=\frac{1}{2}\int \frac{\sqrt{1+u}}{u^2}du-\frac{1}{2}\int \frac{\sqrt{1-u}}{u^2}du.$$

Pas zgjidhjes mirret:

$$I=-\frac{\sqrt{1+e^x}}{e^x}+\frac{1}{2}\ln \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}+C.$$

5. Zbatojmë metodën parciale të integrimit:

$$\int \ln(\sqrt{1-x}+\sqrt{1+x})dx=\begin{vmatrix} \ln(\sqrt{1-x}+\sqrt{1+x})=u \\ \frac{1}{2}\frac{1}{\sqrt{1-x}+\sqrt{1+x})}\left(\frac{1}{\sqrt{1+x}}-\frac{1}{\sqrt{1-x}}\right)dx=du \\ v=x \end{vmatrix}$$

$$=x\ln(\sqrt{1-x}+\sqrt{1+x})-\frac{1}{2}\int \frac{x}{\sqrt{1-x}+\sqrt{1+x}}\left(\frac{1}{\sqrt{1+x)}}-\frac{1}{\sqrt{1-x}}\right)dx$$

$$=x\ln(\sqrt{1-x}+\sqrt{1+x})-\frac{1}{2}\int \frac{x(\sqrt{1-x}-\sqrt{1+x})}{-2x}\frac{dx}{1+x}$$

$$+\frac{1}{2}\int \frac{x(\sqrt{1-x}-\sqrt{1+x})}{-2x}\frac{dx}{\sqrt{1-x}}=x\ln(\sqrt{1-x}+\sqrt{1+x})$$

$$+\frac{1}{4}\int\left(\sqrt{\frac{1-x}{1+x}}-1\right)dx+\frac{1}{4}\int\left(\sqrt{\frac{1+x}{1-x}}-1\right)dx$$

$$= x\ln(\sqrt{1-x}+\sqrt{1+x}) - \frac{x}{2} + \frac{1}{4}\int\left(\sqrt{\frac{1-x}{1+x}}+\sqrt{\frac{1+x}{1-x}}\right)dx$$

$$= x\ln(\sqrt{1-x}+\sqrt{1+x}) - \frac{x}{2} + \frac{1}{2}\arcsin x + C,\ |x|<1.$$

Detyra për ushtrime

Të njehsohen integralet:

1. $\int x^2 e^x \cos x dx.$ **2.** $\int \sqrt{\frac{e^x+1}{e^x-1}}.$ **3.** $\int \frac{dx}{1+e^{\frac{x}{2}}+e^{\frac{x}{3}}+e^{\frac{x}{6}}}.$

4. $\int \frac{e^{4x}}{1+4e^{2x}}dx.$ **5.** $\int \ln^2(x+\sqrt{1+x^2})dx.$ **6.** $\int \frac{\ln x}{\sqrt{(1+x^2)^3}}dx.$

7. $\int x \arccos\frac{1}{x}dx.$ **8.** $\int x\arctan x \ln(1+x^2)dx.$